

Anhang A

Fundamentalsysteme

Auf den folgenden Seiten sind für einige binäre Formen mehrere Tabellen abgedruckt. Spalten entsprechen einem Grad, Zeilen einer Ordnung. Die jeweils erste Tabelle gibt eine Übersicht über die Anzahl der linear unabhängigen Kovarianten in den jeweiligen Ordgraden, die zweite Tabelle gibt Auskunft über die Anzahl der Grundformen in jedem Ordgrad. Ab der Fünfform wurde auch eine Tabelle mit dem Ergebnis der naiven Siebung abgedruckt.

Bis zur binären Achtform ist angegeben, wie die jeweiligen Grundformen durch Überschiebung erzeugt werden können. Die Indizes einer Grundform $K_{i,j}$ entsprechen der Ordnung i und dem Grad j dieser Grundform. Um die Lesbarkeit zu verbessern, wurde die jeweilige binäre Form nur mit f bezeichnet. Zudem wurde, wenn mehrere Grundformen im selben Ordgrad existieren, in ihrer Bezeichnung nicht unterscheiden.

A.1 Elementare Fundamentalsysteme

A.1.1 Zweiform

Linear unabhängige Kovarianten

	0	2	4	6
1		1		
2	1		1	
3		1		1
4	1		1	
5		1		1
6	1		1	

Fundamentalsystem

	0	2
1		1
2	1	

Grundformen

$$f_0 := f$$

$$f_1 := K_{2,0} = (f, f)^2$$

A.1.2 Dreiform

Linear unabhängige Kovarianten

	0	1	2	3	4	5	6
1				1			
2			1				1
3				1		1	
4	1				1		1
5				1		1	
6			1				2

Fundamentalsystem

	0	1	2	3
1				1
2			1	
3				1
4	1			

Grundformen

$$f_0 := f$$

$$f_1 := K_{2,2} = (f, f)^2$$

$$f_2 := K_{3,3} = (f, f_1)^1$$

$$f_3 := K_{4,0} = (f, f_2)^3$$

A.1.3 Vierform

Linear unabhängige
Kovarianten

	0	2	4	6	8	10
1			1			
2	1		1		1	
3	1		1	1	1	
4	1		2		2	1
5	1		2	1	2	1
6	2		2	1	3	1

Fundamental-
system

	0	2	4	6
1			1	
2	1		1	
3	1			1

Grundformen

$f_0 := f$
 $f_1 := K_{2,0} = (f, f)^4$
 $f_2 := K_{2,4} = (f, f)^2$
 $f_3 := K_{3,0} = (f, f_2)^4$
 $f_4 := K_{3,6} = (f, f_2)^1$

A.2 Weitere klassische Fundamentalsysteme

A.2.1 Fünfform

Linear unabhängige Kovarianten

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1						1										
2			1				1				1					
3				1		1		1		1		1				1
4	1				2		1		2		1		2		1	
5		1		1		2		2		3		2		2		2
6			2		1		4		2		4		3		4	
7		1		2		3		4		4		5		4		5
8	2		1		4		3		6		5		7		5	
9		1		3		5		5		7		7		8		8
10			4		3		7		6		10		8		11	
11		2		4		6		8		9		11		11		13
12	3		2		7		7		11		10		15		13	
13		3		5		8		10		13		14		16		17
14			6		6		12		11		17		16		21	
15		3		7		10		13		16		19		20		24
16	4		4		11		11		18		18		24		23	
17		4		8		13		16		20		23		27		29
18	1		9		9		18		18		26		26		33	
19		5		10		15		20		24		29		32		37
20	5		6		16		17		26		27		36		36	

Naive Siebung

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1						1										
2			1				1									
3				1		1				1						
4	1				1		1									
5		1		1				1				-1				
6			1		1		-1		-1		-1		-1		-1	
7		1				1		-1		-3		-2		-3		-2
8	1		1				-2		-3		-4		-6		-5	
9				1		-1		-5		-4		-9		-8		-11
10					-1		-4		-7		-11		-14		-16	
11		1				-4		-6		-14		-16		-25		-25
12	1				-2		-7		-14		-21		-32		-35	
13		1		-2		-4		-15		-20		-39		-45		-59
14			-1		-3		-14		-23		-41		-54		-77	
15				-2		-10		-20		-44		-60		-90		-108
16					-8		-18		-41		-62		-105		-126	
17				-4		-13		-38		-61		-107		-144		-202
18	1		-3		-8		-33		-56		-110		-153		-231	
19				-5		-23		-50		-103		-156		-246		-319
20			-2		-16		-39		-95		-150		-259		-340	

Fundamentalsystem

	0	1	2	3	4	5	6	7	8	9	#	Σ
1						1					1	1
2			1				1				2	3
3				1		1				1	3	6
4	1				1		1				3	9
5		1		1				1			3	12
6			1		1						2	14
7		1				1					2	16
8	1		1								2	18
9				1							1	19
11		1									1	20
12	1										1	21
13		1									1	22
18	1										1	23

Invarianten = 4, Kovarianten = 23

Erzeugung der Grundformen

$$\begin{array}{ll}
f_1 := K_{2,2} = (f, f)^4 & f_{12} := K_{6,2} = (f, f_{10})^3 \\
f_2 := K_{2,6} = (f, f)^2 & f_{13} := K_{6,4} = (f, f_9)^1 \\
f_3 := K_{3,3} = (f, f_1)^2 & f_{14} := K_{7,1} = (f, f_{13})^4 \\
f_4 := K_{3,5} = (f, f_1)^1 & f_{15} := K_{7,5} = (f, f_{12})^1 \\
f_5 := K_{3,9} = (f, f_2)^1 & f_{16} := K_{8,0} = (f, f_{15})^5 \\
f_6 := K_{4,0} = (f, f_4)^5 & f_{17} := K_{8,2} = (f, f_{15})^4 \\
f_7 := K_{4,4} = (f, f_3)^2 & f_{18} := K_{9,3} = (f, f_{17})^2 \\
f_8 := K_{4,6} = (f, f_3)^1 & f_{19} := K_{11,1} = (f, f_9 f_{10})^4 \\
f_9 := K_{5,1} = (f, f_7)^4 & f_{20} := K_{12,0} = (f, f_{10} f_{12})^5 \\
f_{10} := K_{5,3} = (f, f_7)^3 & f_{21} := K_{13,1} = (f, f_1 f_9^2)^4 \\
f_{11} := K_{5,7} = (f, f_7)^1 & f_{22} := K_{18,0} = (f, f_1 f_9^3)^5
\end{array}$$

A.2.2 Sechsform

Linear unabhängige Kovarianten

	0	2	4	6	8	10	12	14	16
1				1					
2	1		1		1		1		
3		1		2	1	1	1	1	
4	2		2	1	3	1	3	1	2
5		2	1	4	2	4	3	4	2
6	3		4	3	6	3	7	4	6
7		4	2	7	5	8	7	9	6
8	4	1	7	5	11	7	13	9	13
9		6	4	12	9	14	13	17	13
10	6	2	11	9	17	13	22	17	23
11		9	7	18	15	23	21	28	24
12	8	4	16	14	26	21	34	28	37
13		13	11	26	23	34	33	43	38
14	10	6	23	21	37	32	49	43	56
15	1	18	16	36	33	49	48	62	57
16	13	9	31	30	51	46	68	62	80
17	1	24	23	48	46	67	67	86	81
18	16	13	41	41	68	63	92	86	109
19	2	31	31	63	62	89	90	115	111
20	20	18	53	54	88	85	120	115	145

Naive Siebung

	0	2	4	6	8	10	12	14	16
1				1					
2	1		1		1				
3		1		1	1		1		
4	1		1	1		1			
5		1	1		1				-1
6	1			2	-1	-1	-1	-2	-1
7		1	1	-1		-1	-4	-2	-4
8		1			-3	-5	-5	-7	-7
9			1	-2	-4	-5	-10	-11	-15
10	1	1	-1	-2	-7	-12	-15	-22	-23
11				-4	-10	-15	-26	-29	-42
12		1	-2	-6	-15	-26	-36	-53	-60
13			-1	-9	-21	-32	-57	-67	-95
14			-4	-12	-30	-52	-71	-109	-134
15	1		-4	-16	-39	-61	-108	-141	-191
16			-7	-23	-51	-94	-136	-202	-262
17			-8	-28	-67	-108	-189	-258	-360
18		-1	-11	-38	-85	-157	-237	-355	-473
19			-14	-45	-109	-180	-315	-444	-622
20		-2	-19	-60	-132	-249	-386	-587	-808

Fundamentalsystem

	0	2	4	6	8	10	12	#	Σ
1				1				1	1
2	1		1		1			3	4
3		1		1	1		1	4	8
4	1		1	1		1		4	12
5		1	1		1			3	15
6	1			2				3	18
7		1	1					2	20
8		1						1	21
9			1					1	22
10	1	1						2	24
12		1						1	25
15	1							1	26

Invarianten = 5, Kovarianten = 26

Erzeugung der Grundformen

$$\begin{aligned}
 f_1 &:= K_{2,0} = (f, f)^6 \\
 f_2 &:= K_{2,4} = (f, f)^4 \\
 f_3 &:= K_{2,8} = (f, f)^2 \\
 f_4 &:= K_{3,2} = (f, f_2)^4 \\
 f_5 &:= K_{3,6} = (f, f_2)^2 \\
 f_6 &:= K_{3,8} = (f, f_2)^1 \\
 f_7 &:= K_{3,12} = (f, f_3)^1 \\
 f_8 &:= K_{4,0} = (f, f_5)^6 \\
 f_9 &:= K_{4,4} = (f, f_4)^2 \\
 f_{10} &:= K_{4,6} = (f, f_4)^1 \\
 f_{11} &:= K_{4,10} = (f, f_5)^1 \\
 f_{12} &:= K_{5,2} = (f, f_9)^4 \\
 f_{13} &:= K_{5,4} = (f, f_9)^3 \\
 f_{14} &:= K_{5,8} = (f, f_9)^1 \\
 f_{15} &:= K_{6,0} = (f, f_2 f_4)^6 \\
 f_{16} &:= K_{6,6} = (f, f_{12})^1 \\
 f_{17} &:= K_{6,6} = (f, f_{13})^2 \\
 f_{18} &:= K_{7,2} = (f, f_{16})^5 \\
 f_{19} &:= K_{7,4} = (f, f_{16})^4 \\
 f_{20} &:= K_{8,2} = (f, f_{19})^4 \\
 f_{21} &:= K_{9,4} = (f, f_{20})^2 \\
 f_{22} &:= K_{10,0} = (f, f_4^3)^6 \\
 f_{23} &:= K_{10,2} = (f, f_{21})^4 \\
 f_{24} &:= K_{12,2} = (f, f_4^2 f_{12})^5 \\
 f_{25} &:= K_{15,0} = (f, f_{18} f_{19})^6
 \end{aligned}$$

A.2.3 Achtform

Linear unabhängige Kovarianten

	0	2	4	6	8	10	12	14	16	18	20	22
1					1							
2	1		1		1		1		1			
3	1		1	1	2	1	2	1	1	1	1	
4	2		3	1	4	2	4	2	4	2	3	1
5	2	1	4	3	6	5	7	5	7	5	6	4
6	4	1	7	5	11	7	13	9	13	10	12	8
7	4	3	10	9	16	14	19	17	21	18	21	17
8	7	4	16	13	25	21	31	26	35	29	35	29
9	8	8	21	22	35	33	45	42	51	48	54	48
10	12	10	32	30	51	48	66	61	77	70	83	74
11	13	17	42	45	69	70	91	90	108	105	118	112
12	20	22	58	61	96	95	128	124	152	147	169	159
13	22	33	75	85	126	133	169	173	205	205	231	226
14	31	42	101	111	168	175	227	230	277	275	314	307
15	36	59	126	150	215	233	294	307	360	369	414	414
16	47	74	165	190	279	299	381	396	471	479	543	543

Naive Siebung

	0	2	4	6	8	10	12	14	16	18	20	22
1					1							
2	1		1		1		1					
3	1		1	1	1	1	1	1		1		
4	1		2	1	1	2	1	1		1		
5	1	1	2	2	1	3		1		-1		-1
6	1	1	2	3	1	1	-2	-1	-3	-4	-3	-4
7	1	2	2	3		-1	-4	-7	-7	-11	-8	-12
8	1	2	2	2	-3	-6	-12	-16	-20	-24	-26	-29
9	1	3	1		-6	-15	-23	-35	-40	-55	-54	-67
10	1	2		-4	-16	-30	-49	-64	-85	-105	-119	-136
11		2	-1	-10	-28	-56	-83	-122	-153	-202	-223	-273
12		1	-5	-20	-53	-96	-151	-210	-279	-357	-424	-500
13			-8	-38	-85	-160	-243	-360	-467	-628	-742	-901
14		-2	-18	-63	-142	-253	-403	-581	-792	-1038	-1283	-1534
15		-4	-28	-101	-215	-399	-622	-936	-1265	-1702	-2097	-2573
16	-1	-7	-49	-155	-334	-601	-973	-1443	-2013	-2674	-3392	-4147

Fundamentalsystem

	0	2	4	6	8	10	12	14	16	18	#	Σ
1					1						1	1
2	1		1		1		1				4	5
3	1		1	1	1	1	1	1		1	8	13
4	1		2	1	1	2	1	1		1	10	23
5	1	1	2	2	1	3		1			11	34
6	1	1	2	3	1	1					9	43
7	1	2	2	3							8	51
8	1	2	2	2							7	58
9	1	3	1								5	63
10	1	2									3	66
11		2									2	68
12		1									1	69

Invarianten = 9, Kovarianten = 69

Erzeugung der Grundformen

$$\begin{array}{ll}
f_1 := K_{2,0} = (f, f)^8 & f_{35} := K_{6,2} = (f, f_{27})^6 \\
f_2 := K_{2,4} = (f, f)^6 & f_{36} := K_{6,4} = (f, f_{25})^4 \\
f_3 := K_{2,8} = (f, f)^4 & f_{37} := K_{6,4} = (f, f_{26})^4 \\
f_4 := K_{2,12} = (f, f)^2 & f_{38} := K_{6,6} = (f, f_{24})^2 \\
f_5 := K_{3,0} = (f, f_3)^8 & f_{39} := K_{6,6} = (f, f_{25})^3 \\
f_6 := K_{3,4} = (f, f_2)^4 & f_{40} := K_{6,6} = (f, f_{26})^3 \\
f_7 := K_{3,6} = (f, f_2)^3 & f_{41} := K_{6,8} = (f, f_{24})^1 \\
f_8 := K_{3,8} = (f, f_2)^2 & f_{42} := K_{6,10} = (f, f_{25})^1 \\
f_9 := K_{3,10} = (f, f_2)^1 & f_{43} := K_{7,0} = (f, f_6^2)^8 \\
f_{10} := K_{3,12} = (f, f_3)^2 & f_{44} := K_{7,2} = (f, f_{38})^6 \\
f_{11} := K_{3,14} = (f, f_3)^1 & f_{45} := K_{7,2} = (f, f_{39})^6 \\
f_{12} := K_{3,18} = (f, f_4)^1 & f_{46} := K_{7,4} = (f, f_{37})^4 \\
f_{13} := K_{4,0} = (f, f_8)^8 & f_{47} := K_{7,4} = (f, f_{38})^5 \\
f_{14} := K_{4,4} = (f, f_6)^4 & f_{48} := K_{7,6} = (f, f_{35})^2 \\
f_{15} := K_{4,4} = (f, f_7)^5 & f_{49} := K_{7,6} = (f, f_{36})^3 \\
f_{16} := K_{4,6} = (f, f_6)^3 & f_{50} := K_{7,6} = (f, f_{37})^3 \\
f_{17} := K_{4,8} = (f, f_6)^2 & f_{51} := K_{8,0} = (f, f_6 f_{14})^8 \\
f_{18} := K_{4,10} = (f, f_6)^1 & f_{52} := K_{8,2} = (f, f_{48})^6 \\
f_{19} := K_{4,10} = (f, f_7)^2 & f_{53} := K_{8,2} = (f, f_{49})^6 \\
f_{20} := K_{4,12} = (f, f_7)^1 & f_{54} := K_{8,4} = (f, f_{46})^4 \\
f_{21} := K_{4,14} = (f, f_8)^1 & f_{55} := K_{8,4} = (f, f_{47})^4 \\
f_{22} := K_{4,18} = (f, f_{10})^1 & f_{56} := K_{8,6} = (f, f_{44})^2 \\
f_{23} := K_{5,0} = (f, f_{17})^8 & f_{57} := K_{8,6} = (f, f_{45})^2 \\
f_{24} := K_{5,2} = (f, f_{16})^6 & f_{58} := K_{9,0} = (f, f_{14} f_{15})^8 \\
f_{25} := K_{5,4} = (f, f_{14})^4 & f_{59} := K_{9,2} = (f, f_{56})^6 \\
f_{26} := K_{5,4} = (f, f_{15})^4 & f_{60} := K_{9,2} = (f, f_{57})^6 \\
f_{27} := K_{5,6} = (f, f_{14})^3 & f_{61} := K_{9,2} = (f, f_{14}^2)^7 \\
f_{28} := K_{5,6} = (f, f_{15})^3 & f_{62} := K_{9,4} = (f, f_{55})^4 \\
f_{29} := K_{5,8} = (f, f_{14})^2 & f_{63} := K_{10,0} = (f, f_{16} f_{24})^8 \\
f_{30} := K_{5,10} = (f, f_{14})^1 & f_{64} := K_{10,2} = (f, f_{14} f_{24})^6 \\
f_{31} := K_{5,10} = (f, f_{15})^1 & f_{65} := K_{10,2} = (f, f_{15} f_{24})^6 \\
f_{32} := K_{5,10} = (f, f_{16})^2 & f_{66} := K_{11,2} = (f, f_{24} f_{25})^6 \\
f_{33} := K_{5,14} = (f, f_{17})^1 & f_{67} := K_{11,2} = (f, f_{25}^2)^7 \\
f_{34} := K_{6,0} = (f, f_{29})^8 & f_{68} := K_{12,2} = (f, f_{25} f_{35})^6
\end{array}$$

A.3 Die Siebenform

Linear unabhängige Kovarianten

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1								1								
2			1				1				1				1	
3				1		1		1		2		1		1		1
4	1				3		1		3		2		3		2	
5		1		2		3		4		4		5		4		5
6			4		2		7		5		8		7		9	
7		3		4		7		9		10		11		13		12
8	4		3		10		9		16		14		19		17	
9		4		10		13		17		21		24		25		29
10			12		13		23		23		34		31		40	
11		8		16		24		31		38		43		49		52
12	10		12		29		33		48		49		65		64	
13		14		26		39		53		62		74		83		91
14	4		30		37		62		68		91		95		116	
15		20		42		62		80		101		116		132		147
16	18		33		70		81		117		129		159		168	
17		31		62		92		122		149		177		200		225
18	13		63		85		137		157		203		223		265	
19		46		88		133		176		216		255		295		326
20	35		71		140		175		241		273		335		363	
21		61		126		185		245		304		360		411		466
22	26		119		173		261		312		400		443		525	
23		85		169		253		334		415		491		567		636
24	62		136		254		330		445		513		626		688	
25		114		224		336		448		551		658		758		855
26	52		207		309		461		560		707		800		940	
27		146		295		440		581		726		860		994		1124
28	97		236		433		565		757		888		1069		1192	
29		189		378		564		750		930		1110		1281		1452
30	92		336		513		759		931		1166		1335		1560	

In der Tabelle auf der übernächsten Seite sind die kleinen Zahlen von VON GALL. Die Unterschiede zwischen Siebung und exakter Berechnung sind durch kursive Zahlen und ■ hervorgehoben.

Vollständige naive Siebung der Siebenform

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1								1								
2			1				1				1					
3				1		1		1		1		1				1
4	1				2		1		2		1				1	
5		1		2		2		2		2						
6			3		2		2		2		-2				-4	
7		3		2		4		2		-3		-4		-6		-8
8	3		3		3		3		-7		-6		-17		-14	
9		3		5		2		-9		-12		-24		-29		-35
10			4		3		-10		-18		-42		-45		-67	
11		5		3		-11		-20		-54		-73		-98		-124
12	6		6		-12		-24		-70		-96		-156		-180	
13		7		-7		-24		-81		-123		-204		-271		-378
14	4		-6		-22		-86		-147		-270		-365		-557	
15		3		-19		-82		-164		-316		-485		-736		-983
16	2		-6		-81		-165		-366		-580		-969		-1301	
17		2		-59		-162		-379		-672		-1165		-1691		-2414
18	9		-42		-138		-386		-713		-1376		-2057		-3134	
19		-11		-111		-345		-742		-1485		-2446		-3818		-5371
20	-1		-73		-304		-699		-1573		-2704		-4570		-6593	
21		-29		-223		-632		-1529		-2919		-5110		-7899		-11617
22	1		-144		-520		-1435		-2926		-5599		-8983		-13974	
23		-56		-385		-1237		-2816		-5736		-9941		-16013		-23813
24	-11		-238		-1004		-2542		-5695		-10400		-17914		-27540	
25		-113		-694		-2157		-5295		-10522		-19023		-31014		-47967
26	-25		-437		-1663		-4705		-10102		-19597		-33482		-54414	
27		-175		-1169		-3853		-9263		-19188		-34994		-59346		-92882
28	-34		-667		-2978		-7967		-18111		-34984		-62902		-102363	
29		-293		-1967		-6502		-16094		-33686		-63887		-109609		-176210
30	-74		-1144		-4821		-13705		-30869		-62751		-113178		-190598	

Naive Siebung

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	#	Σ
1								1									1	1
2			1				1			1							3	4
3				1		1		1		1					1		6	10
4	1				2		1	2		1					1		8	18
5		1		2		2		2		2				■			9	27
6			3		2		2	2					■				9	36
7		3		2		4		2				■					11	47
8	3		3		3		3				■						12	59
9		3		5		2				■							10	69
10			4		3				■								7	76
11		5		3				■									8	84
12	6		6				■										12	96
13		7		■		■											7	103
14	4				■												4	107
15		3		■													3	110
16	2		■														2	112
17		2															2	114
18	9																9	123
19		■															0	
20	■																0	
21																	0	
22	1																1	124

Invarianten = 26, Kovarianten = 124

Fundamentalsystem

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	#	Σ
1								1									1	1
2			1				1			1							3	4
3				1		1		1		1		1				1	6	10
4	1				2		1	2		1					1		8	18
5		1		2		2		2		2				1			10	28
6			3		2		2	2					1				10	38
7		3		2		4		2				1					12	50
8	3		3		3		3			1							13	63
9		3		5		2				1							11	74
10			4		4				1								9	83
11		5		3				1									9	92
12	6		6				1										13	105
13		7		1		1											9	114
14	4				2												6	120
15		3		1													4	124
16	2		3														5	129
17		2															2	131
18	9		0 ₁														9	140
19		1 ₂															1	141
20	1 ₂																1	142
21																	0	
22	2 ₃																2	144
23		1															1	145
24																	0	
25		0 ₁															0	
26	1 ₂																1	146
27																	0	
28																	0	
29																	0	
30	1																1	147

Invarianten = 30₃₃, Kovarianten = 147₁₅₃

Erzeugung der Grundformen

$f_1 := K_{2,2} = (f, f)^6$	$f_{51} := K_{8,0} = (f, f_{48})^7$	$f_{101} := K_{12,2} = (f, f_1 f_4 f_{28})^6$
$f_2 := K_{2,6} = (f, f)^4$	$f_{52} := K_{8,0} = (f, f_1^2 f_4)^7$	$f_{102} := K_{12,2} = (f, f_{19} f_{28})^5$
$f_3 := K_{2,10} = (f, f)^2$	$f_{53} := K_{8,2} = (f, f_{43})^5$	$f_{103} := K_{12,2} = (f, f_{20} f_{28})^5$
$f_4 := K_{3,3} = (f, f_2)^5$	$f_{54} := K_{8,2} = (f, f_{44})^5$	$f_{104} := K_{12,6} = (f, f_{83})^1$
$f_5 := K_{3,5} = (f, f_1)^2$	$f_{55} := K_{8,2} = (f, f_{45})^5$	$f_{105} := K_{13,1} = (f, f_{104})^6$
$f_6 := K_{3,7} = (f, f_1)^1$	$f_{56} := K_{8,4} = (f, f_{41})^3$	$f_{106} := K_{13,1} = (f, f_1 f_{18} f_{19})^6$
$f_7 := K_{3,9} = (f, f_2)^2$	$f_{57} := K_{8,4} = (f, f_{42})^3$	$f_{107} := K_{13,1} = (f, f_1 f_{19}^2)^7$
$f_8 := K_{3,11} = (f, f_2)^1$	$f_{58} := K_{8,4} = (f, f_{43})^4$	$f_{108} := K_{13,1} = (f, f_1 f_{18} f_{20})^6$
$f_9 := K_{3,15} = (f, f_3)^1$	$f_{59} := K_{8,6} = (f, f_{39})^1$	$f_{109} := K_{13,1} = (f, f_4^2 f_{28})^7$
$f_{10} := K_{4,0} = (f, f_6)^7$	$f_{60} := K_{8,6} = (f, f_{40})^1$	$f_{110} := K_{13,1} = (f, f_1 f_{11} f_{28})^7$
$f_{11} := K_{4,4} = (f, f_4)^3$	$f_{61} := K_{8,6} = (f, f_{41})^2$	$f_{111} := K_{13,1} = (f, f_1 f_{12} f_{28})^7$
$f_{12} := K_{4,4} = (f, f_5)^4$	$f_{62} := K_{8,10} = (f, f_{43})^1$	$f_{112} := K_{13,3} = (f, f_{28}^2)^4$
$f_{13} := K_{4,6} = (f, f_4)^2$	$f_{63} := K_{9,1} = (f, f_{59})^6$	$f_{113} := K_{13,5} = (f, f_{98})^2$
$f_{14} := K_{4,8} = (f, f_4)^1$	$f_{64} := K_{9,1} = (f, f_{60})^6$	$f_{114} := K_{14,0} = (f, f_4 f_{18} f_{19})^7$
$f_{15} := K_{4,8} = (f, f_5)^2$	$f_{65} := K_{9,1} = (f, f_{61})^6$	$f_{115} := K_{14,0} = (f, f_4 f_{18} f_{20})^7$
$f_{16} := K_{4,10} = (f, f_5)^1$	$f_{66} := K_{9,3} = (f, f_{56})^4$	$f_{116} := K_{14,0} = (f, f_1 f_{19} f_{28})^7$
$f_{17} := K_{4,14} = (f, f_7)^1$	$f_{67} := K_{9,3} = (f, f_{57})^4$	$f_{117} := K_{14,0} = (f, f_1 f_{19} f_{29})^7$
$f_{18} := K_{5,1} = (f, f_{13})^6$	$f_{68} := K_{9,3} = (f, f_{58})^4$	$f_{118} := K_{14,4} = (f, f_{112})^3$
$f_{19} := K_{5,3} = (f, f_{11})^4$	$f_{69} := K_{9,3} = (f, f_{61})^5$	$f_{119} := K_{14,4} = (f, f_{113})^4$
$f_{20} := K_{5,3} = (f, f_{13})^5$	$f_{70} := K_{9,3} = (f, f_{62})^7$	$f_{120} := K_{15,1} = (f, f_4 f_{18} f_{28})^6$
$f_{21} := K_{5,5} = (f, f_{11})^3$	$f_{71} := K_{9,5} = (f, f_{53})^2$	$f_{121} := K_{15,1} = (f, f_4 f_{19} f_{28})^7$
$f_{22} := K_{5,5} = (f, f_{12})^3$	$f_{72} := K_{9,5} = (f, f_{54})^2$	$f_{122} := K_{15,1} = (f, f_4 f_{20} f_{28})^7$
$f_{23} := K_{5,7} = (f, f_{11})^2$	$f_{73} := K_{9,9} = (f, f_{56})^1$	$f_{123} := K_{15,3} = (f, f_{118})^4$
$f_{24} := K_{5,7} = (f, f_{12})^2$	$f_{74} := K_{10,2} = (f, f_{73})^7$	$f_{124} := K_{16,0} = (f, f_1 f_4 f_{18}^2)^7$
$f_{25} := K_{5,9} = (f, f_{11})^1$	$f_{75} := K_{10,2} = (f, f_4^3)^7$	$f_{125} := K_{16,0} = (f, f_4 f_{28}^2)^7$
$f_{26} := K_{5,9} = (f, f_{12})^1$	$f_{76} := K_{10,2} = (f, f_1^2 f_{18})^5$	$f_{126} := K_{16,2} = (f, f_{19}^3)^7$
$f_{27} := K_{5,13} = (f, f_{15})^1$	$f_{77} := K_{10,2} = (f, f_1^2 f_{19})^6$	$f_{127} := K_{16,2} = (f, f_4 f_{28}^2)^6$
$f_{28} := K_{6,2} = (f, f_{21})^5$	$f_{78} := K_{10,4} = (f, f_{66})^3$	$f_{128} := K_{16,2} = (f, f_4 f_{28} f_{29})^6$
$f_{29} := K_{6,2} = (f, f_{22})^5$	$f_{79} := K_{10,4} = (f, f_{67})^3$	$f_{129} := K_{17,1} = (f, f_{19}^2 f_{28})^7$
$f_{30} := K_{6,2} = (f, f_{23})^6$	$f_{80} := K_{10,4} = (f, f_{68})^3$	$f_{130} := K_{17,1} = (f, f_{11} f_{28}^2)^7$
$f_{31} := K_{6,4} = (f, f_{19})^3$	$f_{81} := K_{10,4} = (f, f_{69})^3$	$f_{131} := K_{18,0} = (f, f_1 f_{18}^2 f_{19})^7$
$f_{32} := K_{6,4} = (f, f_{20})^3$	$f_{82} := K_{10,8} = (f, f_{66})^1$	$f_{132} := K_{18,0} = (f, f_1 f_{18}^2 f_{20})^7$
$f_{33} := K_{6,6} = (f, f_{18})^1$	$f_{83} := K_{11,1} = (f, f_{82})^7$	$f_{133} := K_{18,0} = (f, f_{19} f_{28}^2)^7$
$f_{34} := K_{6,6} = (f, f_{19})^2$	$f_{84} := K_{11,1} = (f, f_1 f_4 f_{18})^6$	$f_{134} := K_{18,0} = (f, f_{20} f_{28}^2)^7$
$f_{35} := K_{6,8} = (f, f_{19})^1$	$f_{85} := K_{11,1} = (f, f_1 f_4 f_{19})^7$	$f_{135} := K_{18,0} = (f, f_{19} f_{28} f_{29})^7$
$f_{36} := K_{6,8} = (f, f_{21})^2$	$f_{86} := K_{11,1} = (f, f_{19}^2)^6$	$f_{136} := K_{18,0} = (f, f_{20} f_{28} f_{29})^7$
$f_{37} := K_{6,12} = (f, f_{23})^1$	$f_{87} := K_{11,1} = (f, f_{18} f_{21})^6$	$f_{137} := K_{18,0} = (f, f_{19} f_{29}^2)^7$
$f_{38} := K_{7,1} = (f, f_{33})^6$	$f_{88} := K_{11,3} = (f, f_{79})^4$	$f_{138} := K_{18,0} = (f, f_{20} f_{29}^2)^7$
$f_{39} := K_{7,1} = (f, f_{34})^6$	$f_{89} := K_{11,3} = (f, f_{80})^4$	$f_{139} := K_{18,0} = (f, f_{19} f_{28} f_{30})^7$
$f_{40} := K_{7,1} = (f, f_{36})^7$	$f_{90} := K_{11,3} = (f, f_{81})^4$	$f_{140} := K_{19,1} = (f, f_{28}^3)^6$
$f_{41} := K_{7,3} = (f, f_{31})^4$	$f_{91} := K_{11,7} = (f, f_{74})^1$	$f_{141} := K_{20,0} = (f, f_{21} f_{40}^2)^7$
$f_{42} := K_{7,3} = (f, f_{32})^4$	$f_{92} := K_{12,0} = (f, f_{91})^7$	$f_{142} := K_{22,0} = (f, f_4 f_{18} f_{28} f_{40})^7$
$f_{43} := K_{7,5} = (f, f_{28})^2$	$f_{93} := K_{12,0} = (f, f_1^3 f_{18})^7$	$f_{143} := K_{22,0} = (f, f_1 f_{28}^2 f_{40})^7$
$f_{44} := K_{7,5} = (f, f_{29})^2$	$f_{94} := K_{12,0} = (f, f_4^2 f_{18})^7$	$f_{144} := K_{23,1} = (f, f_{40}^2 f_{58})^6$
$f_{45} := K_{7,5} = (f, f_{30})^2$	$f_{95} := K_{12,0} = (f, f_1 f_{11} f_{18})^7$	$f_{145} := K_{26,0} = (f, f_{28}^3 f_{40})^7$
$f_{46} := K_{7,5} = (f, f_{31})^3$	$f_{96} := K_{12,0} = (f, f_1 f_4 f_{28})^7$	
$f_{47} := K_{7,7} = (f, f_{28})^1$	$f_{97} := K_{12,0} = (f, f_{21} f_{28})^7$	
$f_{48} := K_{7,7} = (f, f_{29})^1$	$f_{98} := K_{12,2} = (f, f_{91})^6$	
$f_{49} := K_{7,11} = (f, f_{34})^1$	$f_{99} := K_{12,2} = (f, f_4^2 f_{19})^7$	
$f_{50} := K_{8,0} = (f, f_{47})^7$	$f_{100} := K_{12,2} = (f, f_4^2 f_{20})^7$	

Die Invariante I_{30} konnte nur nachgewiesen, aber nicht explizit berechnet werden (s. Kapitel 6.8.9).

A.4 Hohe Formen

A.4.1 Neunform

Fundamentalsystem bis Ordnung 20

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	#	Σ
1																								1	1
2			1				1				1				1									4	5
3				1		1		1		2		1		1		1		1				1		10	15
4	2				2		2		3		2		2		2		1		1				1	18	33
5		1		3		4		4		3		4		2		3				3				25	58
6			4		4		6		6		3		4				1							28	86
7		4		7		8		7		6		1		1										34	120
8	5		8		10		10		4		2													39	159
9		9		14		10		7		1														41	200
10	5		15		15		3		1															39	239
11		17		16		7		1																41	280
12	14		23		4		1																	42	322
13		25		10		1																		36	358
14	17		13		1						\square		?											31	389
15		26		1		?		?		?		?												27	416
16	21		3		?		?		?		?													24	440
17		7		\square																				7	447
18	(25)																							25	472
19																								0	
20	(2)																							2	474

Invarianten = 91, Kovarianten = 474

Siebung

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	#	Σ
1										1														1	1
2			1				1				1				1									4	5
3				1		1		1		2		1		1		1		1				1		10	15
4	2				2		2		3		2		2		2		1		1				1	18	33
5		1		3		4		4		3		4		2		2				\blacksquare				23	56
6			4		4		6		6		3		3				\blacksquare							26	82
7		4		7		8		7		5		\blacksquare		\blacksquare										31	113
8	5		8		10		10		2		\blacksquare													35	148
9		9		14		10		2		\blacksquare														35	183
10	5		15		14		\blacksquare		\blacksquare															34	216
11		17		16		\blacksquare		\blacksquare																33	250
12	14		23		\blacksquare		\blacksquare																	37	287
13		25		\blacksquare		\blacksquare																		25	312
14	17		9		\blacksquare																			26	338
15		26		\blacksquare																				26	364
16	21		\blacksquare																					21	385
17		5																						5	390
18	25																							25	415

Invarianten = 89, Kovarianten = 415

Anmerkung: Bis Ordgrad (14,10) erfolgt die Berechnung des Fundamentalsystems mit dem Programm **kovariante** auf „normale“ Weise. Wegen des zu großen Speicheraufwandes brach das Programm ab (\square). Ein Neustart mit der zusätzlichen Option **-t 2** (siehe *Schrumpfgesetz* auf Seite 205) ermöglichte es, die Berechnung bis Ordgrad (17,1) fortzusetzen. Frühere Ordgrade, in denen theoretisch noch weitere Grundformen sein könnten, wurden mit einem Fragezeichen „?“ gekennzeichnet. Die Werte für die Invarianten in Ordnung 18 und 20 wurde mit dem Programm **invariante** berechnet. Nur die Anzahl, nicht aber die Invarianten selber konnten berechnet werden.

A.4.2 Zehnform

Fundamentalsystem bis Grad 12

	0	2	4	6	8	10	12	14	16	18	20	22	24	26	#	Σ
1						1									1	1
2	1		1		1		1		1						5	6
3		1		2	1	1	2	1	1	1	1		1		12	18
4	1		3	1	3	3	2	3	1	2	1	1		1	22	40
5		3	3	4	5	4	5	2	4		2				32	72
6	4	2	5	8	6	8	2	4		1					40	112
7		7	10	8	12	2	4		1						44	156
8	5	8	11	15	4	7		1							51	207
9	5	13	19	8	7		1								53	260
10	8	20	13	13		1									55	315
11	8	18	21		1										48	363
12	12	30	1	2			\square								45	408

Invarianten = 44, Kovarianten = 408

Siebung

	0	2	4	6	8	10	12	14	16	18	20	22	24	26	#	Σ
1						1									1	1
2	1		1		1		1		1						5	6
3		1		2	1	1	2	1	1	1	1		1		12	18
4	1		3	1	3	3	2	3	1	2	1	1		1	22	40
5		3	3	4	5	4	5	2	4		1				31	71
6	4	2	5	8	6	8	2	3							38	109
7		7	10	8	12	2	3								42	151
8	5	8	11	15	4	5									48	199
9	5	13	19	8	4										49	248
10	8	20	12	10											50	298
11	8	18	21												47	345
12	12	30													42	387
13	15	16													31	418
14	13	17													30	448
15	19														19	467
16	5														5	472
17	3														3	475

Invarianten = 99, Kovarianten = 475

A.4.3 Elfform

Fundamentalsystem bis Grad 10

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	#	Σ
1												1																				1	1
2			1				1				1				1				1													5	6
3				1		1		1		2		2		1		2		1		1		1		1				1				15	21
4	2				3		2		4		3		4		3		3		3		1		2		1		1				1	33	54
5		2		4		5		7		7		6		7		5		5		3		3				2						56	110
6			7		7		11		12		11		11		5		7		1		2				1							75	185
7		8		12		18		18		18		15		6		4		1		1					1							102	287
8	10		17		25		28		21		20		3		3		1		1					1								130	417
9		21		37		37		31		18		3		3		1		1					1									153	570
10	13		45		54		31		16		3		3		1		□															166	736

Invarianten = 25, Kovarianten = 736

Siebung

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	#	Σ
1												1																			1	1	
2			1			1				1				1				1													5	6	
3				1		1		1		2		2		1		2		1		1		1		1				1			15	21	
4	2				3		2		4		3		4		3		3		3		1		2		1		1			1	33	54	
5		2		4		5		7		7		6		7		5		4		3		2									52	106	
6			7		7		11		12		11		11		4		5														68	174	
7		8		12		18		18		17		14		3																	90	264	
8	10		17		25		28		19		16																				115	379	
9		21		37		37		24		7																					126	505	
10	13		45		52		19																								129	634	
11		56		66		27																									149	783	
12	49		92		22																										163	946	
13		107		49																												156	1102
14	84		82																													166	1268
15		149																														149	1417
16	132																															132	1549
17		78																														78	1627
18	192																															192	1819
19																																0	
20	12																															12	1831

Invarianten = 494, Kovarianten = 1831

A.4.4 Zwölfform

Fundamentalsystem bis Grad 10

	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	#	Σ
1							1												1	1
2	1		1		1		1		1		1								6	7
3	1		1	1	2	1	2	2	1	2	1	1	1	1		1			18	25
4	2		3	2	4	3	4	4	3	4	2	3	1	2	1	1		1	40	65
5	2	2	5	6	7	8	6	9	5	6	3	5	1	2		1			68	133
6	4	4	9	11	12	14	10	12	3	6	1	1							87	220
7	5	10	15	20	18	21	9	9	1	1									109	329
8	7	16	24	29	21	21	1	1	1										121	450
9	9	28	33	37	15	1	1	1											125	575
10	14	39	41	30	1	\square													125	700

Invarianten = 45, Kovarianten = 700

Siebung

	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	#	Σ
1							1												1	1
2	1		1		1		1		1		1								6	7
3	1		1	1	2	1	2	2	1	2	1	1	1	1		1			18	25
4	2		3	2	4	3	4	4	3	4	2	3	1	2	1	1		1	40	65
5	2	2	5	6	7	8	6	9	5	6	3	4	1	1					65	130
6	4	4	9	11	12	14	10	12	3	5									84	214
7	5	10	15	20	18	21	9	8											106	320
8	7	16	24	29	21	21													118	438
9	9	28	33	37	15														122	560
10	14	39	41	30															124	684
11	15	53	40																108	792
12	19	56	7																82	874
13	18	44																	62	936
14	12																		12	948

Invarianten = 109, Kovarianten = 948

A.4.5 Dreizehnform

Fundamentalsystem bis Ordnung 9

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	#	Σ
1														1																									1	1	
2			1				1				1			1				1					1																6	7	
3				1		1		1		2		2		2		2		2		1		2		1		1		1		1				1					21	28	
4	2				4		2		5		4		5		5		5		4		4		4		2		3		1		2		1		1				1	55	83
5		3		5		7		9		11		11		10		12		8		10		6		6		4		4		1		2				1			110	193	
6			11		11		18		20		20		24		17		18		9		11		2		5				1										167	360	
7		14		21		32		36		38		37		34		16		13		1		1		1															244	604	
8	19		33		51		62		56		61		34		17		2		1				1																337	941	
9		45		82		94		93		75		□																											389	1330	

Invarianten = 21, Kovarianten = 1330

A.4.6 Siebzehnform

Fundamentalsystem bis Ordnung 7

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	#	Σ
1																		1																																			1	1				
2			1				1				1				1				1				1				1				1																								8	9		
3				1		1		1		2		2		2		3		3		2		3		2		2		2		2		1		2		1		1		1				1												36	45	
4	3				5		3		7		5		8		7		9		8		8		8		8		7		6		7		4		5		3		4		2		3		1		2		1		1			1	126	171		
5		4		8		12		15		18		20		23		24		22		25		21		22		18		19		14		15		10		10		7		7		3		4		2		2		1		1			1	328	499	
6	1		21		23		38		42		49		56		56		61		52		54		39		43		24		30		12		16		5		7		1		3													633	1132			
7		31		55		81		97		112		121		125		121		116		84		70		38		22		1		3																									1077	2209		

Invarianten = 4, Kovarianten = 2209